Two way classification:-

Randomized Block Design (RBD):-

In two way classification of ANOVA we considered one classification along column wise and other along row wise.

Working Procedure:-

step 1: State H_0 and H_1

step 2: Find N

step 3: Find T

step 4: Find $\frac{T^2}{N}$.

Adv: * We can test two set of hypothesis on same data.
* More efficient than CRD.

Disadv: * causing heterogenity
* shape should be rectangular

step 5:- Find $TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \dots - \frac{T^2}{N}$.

step 6: Find $SSC = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma x_2)^2}{N_1} + \frac{(\Sigma x_3)^2}{N_1} + \dots - \frac{T^2}{N}$.

step 7: Find $SSR = \frac{(\Sigma y_1)^2}{N_2} + \frac{(\Sigma y_2)^2}{N_2} + \frac{(\Sigma y_3)^2}{N_2} + \dots - \frac{T^2}{N}$.

N_1 - No. of elements in each column.

N_2 - No. of elements in each rows.

step 8: Find $SSE = TSS - SSC - SSR$.

step 9: ANOVA TABLE

Source of Variation	Sum of squares	DoF	Mean Squares	Variance ratio	Table Value.
Between Column	SSC	$c-1$	$MSC = \frac{SSC}{c-1}$	$F_c = \frac{MSC}{MSE}$ (or) $F_c = \frac{MSE}{MSC}$	
Between rows	SSR	$r-1$	$MSR = \frac{SSR}{r-1}$	$F_R = \frac{MSR}{MSE}$ (or) $F_R = \frac{MSE}{MSR}$	
Error	SSE	$(c-1)\times(r-1)$	$MSE = \frac{SSE}{(c-1)(r-1)}$		

Step 10: Conclusion:

If Fcal < Ftab, we accept Ho.

If Fcal > Ftab, we reject Ho.

Problems:-

5. Perform two way (classification) ANOVA TABLE, the data given below.

Plots of Land	Treatments			
	A	B	C	D
I	38	40	41	39
II	45	40	49	36
III	40	38	42	42

Use coding method, subtract 40 from the given numbers.

Sol:-

Origin = 40.

	A	B	C	D
I	-2	0	1	-1
II	5	2	9	-4
III	0	-2	2	2

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	-2	0	1	-1	$\Sigma y_1 = -2$	4	0	1	1
Y_2	5	2	9	-4	$\Sigma y_2 = 12$	25	4	81	16
Y_3	0	-2	2	2	$\Sigma y_3 =$	0	4	4	4
Total	$\Sigma x_1 = 3$	$\Sigma x_2 = 0$	$\Sigma x_3 = 12$	$\Sigma x_4 = -3$	(12)	$\Sigma x_1^2 = 29$	$\Sigma x_2^2 = 8$	$\Sigma x_3^2 = 86$	$\Sigma x_4^2 = 21$

Step 1:

H_0 - There's no significant difference b/w column means or the row means.

H_1 - There is significant difference b/w column means or the row means.

Step 2:

$N = 12$.

Step 3:

$T = 12$.

Step 4:

$\frac{T^2}{N} = 12$.

Step 5:

$$TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - \frac{T^2}{N}.$$

$$= 29 + 8 + 86 + 21 - \frac{12^2}{12}.$$

$$TSS = 132$$

Step 6:

$$SSC = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma x_2)^2}{N_1} + \frac{(\Sigma x_3)^2}{N_1} + \frac{(\Sigma x_4)^2}{N_1} - \frac{T^2}{N}.$$

$$= \frac{3^2}{3} + \frac{0^2}{3} + \frac{12^2}{3} + \frac{(-3)^2}{3} - 12$$

$$SSC = 42.$$

Step 7:

$$SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}.$$

$$= \frac{(-2)^2}{4} + \frac{(12)^2}{4} + \frac{2^2}{4} - 12$$

$$SSR = 26$$

Step 8:

$$SSE = TSS - SSC - SSR = 132 - 42 - 26 = 64.$$

Step 9: ANOVA TABLE:

Source of Variation	Sum of squares	DOF	Mean Squares	variance ratio	Table Value.
Between Columns	SSC = 42	c−1 = 4−1 = 3	MSC = 42/3 = 14	$F_c = \frac{MSC}{MSE} = \frac{14}{10.67}$	$F_t(3,6) = 4.76$
Between rows	SSR = 26	r−1 = 3−1 = 2	MSR = 26/2 = 13	$F_R = \frac{MSR}{MSE} = \frac{13}{10.67} = 1.218$	$F_R = (2,6) = 5.14$
Error	SSE = 64	(r−1) × (c−1) = 3×2 = 6	MSE = 64/6 = 10.67		

Step 10: conclusion:

Here F_c cal < F_c tab

F_R cal < F_R tab.

∴ In both case, we accept Ho.

There's no significant difference between column means or the row means.

6. An Experiment was designed study the performance of 4 different detergents for cleaning fuel injectors, the following cleanliness reading where obtained, with the specially designed equipment for 12 tanks of gas distributed over three different models of engineers. For the below table, perform the ANOVA and test at 0.01 level of significance whether there are differences in the detergents are in the engines.

Detergent	Engine 1	Engine 2	Engine 3	Total
A	45	43	51	139
B	47	46	52	145
C	48	50	55	153
D	42	37	49	128
Total	182	176	207	565

Sol:-

Origin = 50.

	X_1	X_2	X_3	Total	X_1^2	X_2^2	X_3^2
y_1	-5	-7	1	$\Sigma y_1 = -11$	25	49	1
y_2	-3	-4	2	$\Sigma y_2 = -5$	9	16	4
y_3	-2	0	5	$\Sigma y_3 = 3$	4	0	25
y_4	-8	-13	-1	$\Sigma y_4 = -22$	64	169	1
Total	$\Sigma x_1 = -18$	$\Sigma x_2 = -24$	$\Sigma x_3 = 7$	(-35)	$\Sigma x_1^2 = 102$	$\Sigma x_2^2 = 234$	$\Sigma x_3^2 = 31$

Step 1:

H_0 : There's no significant difference b/w column means or the row means.

H_1 : There is significant difference b/w column means or the row means.

Step 2: $N = 12$

Step 3: $T = -35$

Step 4: $\frac{T^2}{N} = \frac{(-35)^2}{12} = 102.08.$

Step 5:-

$$TSS = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 - \frac{T^2}{N}$$

$$= 102 + 234 + 31 - 102.08$$

$$= 367 - 102.08$$

$$TSS = 264.92$$

Step 6:-

$$SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} - \frac{T^2}{N}$$

$$= \frac{(-18)^2}{4} + \frac{(-24)^2}{4} + \frac{7^2}{4} - 102.08$$

$$= \frac{324}{4} + \frac{576}{4} + \frac{49}{4} - 102.08$$

$$= 81 + 144 + 12.25 - 102.08$$

$$= 237.25 - 102.08$$

$$SSC = 135.17$$

Step 7:-

$$SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{(11)^2}{3} + \frac{(-5)^2}{3} + \frac{3^2}{3} + \frac{(-22)^2}{3} - 102.08$$

$$= \frac{121}{3} + \frac{25}{3} + \frac{9}{3} + \frac{484}{3} - 102.08$$

$$= 40.33 + 8.33 + 3 + 161.33 - 102.08$$

$$= 212.99 - 102.08$$

$$SSR = 110.91$$

Step 8:-

$$SSE = TSS - SSC - SSR$$

$$= 264.92 - 133.17 - 110.91$$

$$SSE = 18.84$$

Step 9:- ANOVA TABLE:-

Source of variation	Sum of squares	DOF	Mean Squares	Variance ratio	Table value.
Between column	SSC = 135.17	C-1 = 3-1 = 2	$MSC = \frac{SSC}{C-1} = \frac{135.17}{2}$ MSC = 67.58	$F_C = \frac{MSC}{MSE} = \frac{67.58}{3.14}$ $F_C = 21.52$	$F_C(2,6) = 10.9$
Between rows	SSR = 110.91	r-1 = 4-1 = 3	$MSR = \frac{SSR}{r-1} = \frac{110.91}{3}$ MSR = 36.97	$F_R = \frac{MSR}{MSE} = \frac{36.97}{3.14}$ $F_R = 11.77$	$F_R(3,6) = 9.78$
Error	SSE = 18.84	(c-1) x (r-1) = 6	$MSE = \frac{SSE}{(c-1)(r-1)} = \frac{18.84}{6}$ MSE = 3.14		

Step 10:- conclusion:-

Here $F_{cal} > F_c$ tab, F_R cal $> F_R$ tab.

∴ In bothe cases we reject H_0.

7. Five doctors, each test give treatments for a certain disease and observe the numbers of days each patient takes to recover. The results are as follows given recovery time in days.

Doctors	Treatments				
	1	2	3	4	5
Jebasingh	10	14	23	19	20
Niranjan	11	15	24	17	21
Deivanai	9	11	20	16	19
Sathyapriya	8	13	17	17	20
Kanimozhi	12	15	19	15	22

Discuss the difference b/w (i) Doctors and (ii) treatments.

Sol:-

Origin = 15.

Σx: (10-15)

	X_1	X_2	X_3	X_4	X_5
Y_1	-5	-1	8	4	5
Y_2	-4	0	9	2	6
Y_3	-6	-4	5	1	4
Y_4	-7	-2	2	2	5
Y_5	-3	0	4	0	7

	X_1	X_2	X_3	X_4	X_5	Total	X_1^2	X_2^2	X_3^2	X_4^2	X_5^2
Y_1	-5	-1	8	4	5	11	25	1	64	16	25
Y_2	-4	0	9	2	6	13	16	0	81	4	36
Y_3	-6	-4	5	1	4	0	36	16	25	1	16
Y_4	-7	-2	2	2	5	0	49	4	4	4	25
Y_5	-3	0	4	0	7	8	9	0	16	0	49
Total	ΣX_1 -25	ΣX_2 -7	ΣX_3 28	ΣX_4 9	ΣX_5 27	Total (32)	ΣX_1^2 135	ΣX_2^2 21	ΣX_3^2 190	ΣX_4^2 25	ΣX_5^2 151

Step 1:

H_0 : There's no significant difference between doctors and treatments.

H_1 : There is significant difference b/w doctors and treatments.

step 2:

$N = 25$.

Step 3:

$T = 32.$

Step 4:-

$$\frac{T^2}{N} = \frac{(32)^2}{25} = \frac{1024}{25} = 40.96.$$

Step 5:

$$TSS = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \Sigma X_4^2 + \Sigma X_5^2 - \frac{T^2}{N}.$$

$$= 135 + 21 + 190 + 25 + 151 - 40.96.$$

$$= 522 - 40.96$$

$$TSS = 481.04$$

Step 6:-

$$SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} + \frac{(\Sigma X_5)^2}{N_1} - \frac{T^2}{N}.$$

$$= \frac{(-25)^2}{5} + \frac{(-7)^2}{5} + \frac{(26)^2}{5} + \frac{9^2}{5} + \frac{27^2}{5} - 40.96$$

$$= 125 + 9.8 + 156.8 + 16.2 + 145.8 - 40.96$$

$$SSC = 412.64$$

Step 7:

$$SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} + \frac{(\Sigma Y_4)^2}{N_2} + \frac{(\Sigma Y_5)^2}{N_2} - \frac{T^2}{N}.$$

$$= \frac{11^2}{5} + \frac{13^2}{5} + \frac{0^2}{5} + \frac{0^2}{5} + \frac{8^2}{5} - 40.96$$

$$= 24.2 + 33.8 + 12.8 - 40.96$$

$$SSR = 29.84$$

Step 8:-

$$SSE = TSS - SSC - SSR$$

$$= 481.04 - 412.64 - 29.84$$

$$SSE = 38.56$$

Step 9:- ANOVA TABLE:-

Source of variation	Sum of squares	DoF	Mean square	Variance ratio	Table Value
Between Column	SSC = 412.64	c-1 = 5-1 = 4	MSC = SSC/(c-1) = 412.64/4 MSC = 103.16	Fc = MSC/MSE = 103.16/2.41 Fc = 42.8	Fc(4,16) = 3.01
Between rows	SSR = 29.84	r-1 = 5-1 = 4	MSR = SSR/(r-1) = 29.84/4 MSR = 7.46	FR = MSR/MSE = 7.46/2.41 FR = 3.09.	FR = (4,16) = 3.01
Errors	SSE = 38.56	(c-1) x (r-1) = 4x4 = 16	MSE = SSE/((c-1)x(r-1)) = 38.56/16 MSE = 2.41.		

Step 10: conclusion:-

Here F_c cal > F_c Tab, F_R cal > F_R Tab.

∴ In the bothe cases, we reject H_0.

H/w

8. The following table gives monthly sales (in thousand rupees) of a certain firm in the three states by its four salesman.

States	Salesman I	II	III	IV
A	6	5	3	8
B	8	9	6	5
C	10	7	8	7

Setup the analysis of variance table and test whether there is any significant difference (i) b/w sales by the firm salesman and (ii) b/w sales in the three states.

Sol:-

Origin = 2.

	X_1	X_2	X_3	X_4	Total	X_1^2	X_2^2	X_3^2	X_4^2
Y_1	1	0	-2	3	$\Sigma Y_1 = 2$	1	0	4	9
Y_2	3	4	1	0	$\Sigma Y_2 = 8$	9	16	1	0
Y_3	5	2	3	2	$\Sigma Y_3 = 12$	25	4	9	4
Total	$\Sigma X_1 = 9$	$\Sigma X_2 = 6$	$\Sigma X_3 = 2$	$\Sigma X_4 = 5$	$T = 22$	$\Sigma X_1^2 = 35$	$\Sigma X_2^2 = 20$	$\Sigma X_3^2 = 14$	$\Sigma X_4^2 = 13.$

Step 1:

H_0 - There's no significant difference in states and stdesman.

H_1 - There's significant difference in states and salesman

step 2: $N = 12.$

step 3: $T = 22.$

Step 4: $\frac{T^2}{N} = \frac{22^2}{12} = 40.32.$

step 5:-

$$TSS = \Sigma X_1^2 + \Sigma X_2^2 + \Sigma X_3^2 + \Sigma X_4^2 - \frac{T^2}{N}$$

$$= 35 + 20 + 14 + 13 - 40.33$$

$$TSS = 41.67$$

Step 6:-

$$SSC = \frac{(\Sigma X_1)^2}{N_1} + \frac{(\Sigma X_2)^2}{N_1} + \frac{(\Sigma X_3)^2}{N_1} + \frac{(\Sigma X_4)^2}{N_1} - \frac{T^2}{N}.$$

$$= \frac{9^2}{3} + \frac{6^2}{3} + \frac{2^2}{3} + \frac{5^2}{3} - 40.32$$

$$SSC = 8.34$$

Step 7:

$$SSR = \frac{(\Sigma Y_1)^2}{N_2} + \frac{(\Sigma Y_2)^2}{N_2} + \frac{(\Sigma Y_3)^2}{N_2} - \frac{T^2}{N}$$

$$= \frac{2^2}{4} + \frac{8^2}{4} + \frac{12^2}{4} - 40.32$$

$$SSR = 12.67$$

Step 8:-

$$SSE = TSS - SSC - SSR$$

$$= 41.67 - 8.34 - 12.67.$$

$$SSE = 20.66$$

Step 9:- ANOVA Table:-

Source of variation	Sum of squares	D.F	Mean squares	Variance ratio	Table Value
Between column	SSC = 8.34	c-1 = 4-1 = 3	$MSC = \frac{SSC}{c-1} = \frac{8.34}{3}$ MSC = 2.78	$F_C = \frac{MSE}{MSC} = \frac{3.44}{2.78}$ $F_C = 1.24$	$F_c(6,3) = 8.94$
Between rows	SSR = 12.67	r-1 = 3-1 = 2	$MSR = \frac{SSR}{r-1} = \frac{12.67}{2}$ MSR = 6.335	$F_R = \frac{MSR}{MSE} = \frac{6.335}{3.44}$ $F_R = 1.84$	$F_R(2,6) = 5.14$
Error	SSE = 20.66	(r-1)(c-1) = 3×2 = 6	$MSE = \frac{SSE}{(r-1)(c-1)} = \frac{20.66}{6}$ MSE = 3.44		

Step 10:- Conclusion:-

Here F_C cal < F_C tab, F_R cal < F_R tab.

∴ In both cases we accept H_0.